

# SAT Subject Physics Facts & Formulas

This document is a concise but comprehensive guide to the facts and formulas typically used in the material covered by the SAT Subject physics test. The test is designed to determine how well you have mastered the physics concepts taught in a typical one-year college-prep high school course.

This guide is mainly intended as a reference, as opposed to a full tutorial (which would probably be book-length), and so the explanatory material is pretty brief. You can use the guide as a simple formula reference, or as a quick review of the material that you've already studied elsewhere. Either way, good luck on your Subject Test!

## Math Stuff

Although this guide is for the SAT Subject test in Physics, you'll need to know quite a bit of math. If you're thinking that you'll just use your calculator to do the math, don't forget that *calculators are not allowed on the SAT Subject Physics test*. Here is a summary of the really important math facts and formulas.

### Exponents

$$\begin{array}{lll} x^a \cdot x^b = x^{a+b} & x^a/x^b = x^{a-b} & 1/x^b = x^{-b} \\ (x^a)^b = x^{a \cdot b} & (xy)^a = x^a \cdot y^a & (-1)^n = \begin{cases} +1, & \text{if } n \text{ is even;} \\ -1, & \text{if } n \text{ is odd.} \end{cases} \\ x^0 = 1 & \sqrt{xy} = \sqrt{x} \cdot \sqrt{y} & \end{array}$$

### Scientific Notation

Scientific notation is a short-hand form to write numbers which would have a lot of zeros when written as decimals. For example, instead of writing 1230000, you can just write  $1.23 \times 1000000$ , or  $1.23 \times 10^6$ . The familiar powers of ten include:

$$10^{-3} = 0.001, 10^{-2} = 0.01, 10^{-1} = 0.1, 10^0 = 1, 10^1 = 10, 10^2 = 100, 10^3 = 1000.$$

To go from scientific notation to a plain decimal number, move the decimal to the right or left according to the sign of the exponent, putting a zero down when you have no other digits there. For example, for  $3.7 \times 10^{12}$ , move the decimal right 12 places and add 11 zeros. Move the decimal to the left for a negative exponent.

$$\begin{array}{l} \text{11 zeros} \\ \overbrace{37\,000000000000} = 3.7 \times 10^{12} \\ \underbrace{.0000000000} 23 = 2.3 \times 10^{-11} \\ \text{10 zeros} \end{array}$$

To go from a plain decimal number to scientific notation, just move the decimal to the right or left (counting how many places you move) until there is only one digit to the left of the decimal point, then add " $\times 10^n$ " where  $n$  is the number of places you moved the decimal point (positive if you went left and negative if you went right).

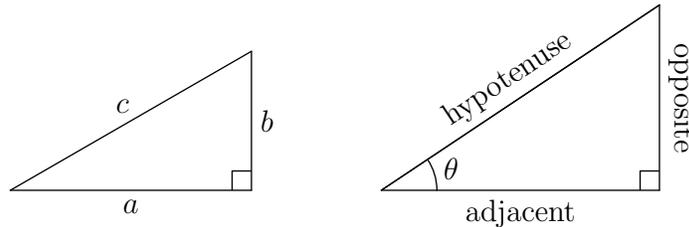
# SAT Subject Physics Facts & Formulas

## Basic Metric Prefixes

Common powers of ten (both positive and negative) have names that come before the metric unit of measurement, i.e., they are prefixes. The most typically used ones are given below.

Prefix	Symbol	Power of Ten	Common Example
nano	n	$10^{-9}$	nanometer
micro	$\mu$	$10^{-6}$	microsecond
milli	m	$10^{-3}$	milligram
centi	c	$10^{-2}$	centimeter
kilo	k	$10^3$	kilogram
mega	M	$10^6$	megawatt

## Basic Trigonometry



In the first triangle above,

$$a^2 + b^2 = c^2 \quad (\text{pythagorean theorem})$$

Referring to the second triangle, there are three important functions which are defined for angles in a right triangle:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

“SOH”

“CAH”

“TOA”

(the last line above shows a mnemonic to remember these functions: “SOH-CAH-TOA”)

An important relationship to remember which works for any angle  $\theta$  is:

$$\sin^2 \theta + \cos^2 \theta = 1.$$

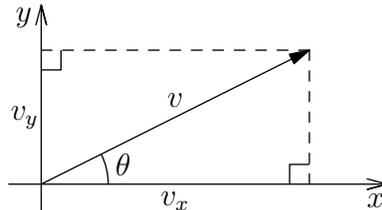
## Vectors

Many important quantities in physics are represented by *vectors*, which specify both a number (the length of the vector) along with a direction (where the vector points). In contrast, *scalars* are simple numbers without a direction.

## SAT Subject Physics Facts & Formulas

For example, velocity is a vector (represented by a boldface  $\mathbf{v}$ ) and is given by a number (say, 50 m/sec) along with a direction (say,  $30^\circ$  north of east). Mass ( $m$ ) is just a number (say, 80 kg), for which a direction doesn't make any sense, so it is a scalar.

We can define *components* of a vector as the projection (or “shadow”) of the vector on the  $x$  and  $y$  axes, as in the figure below.



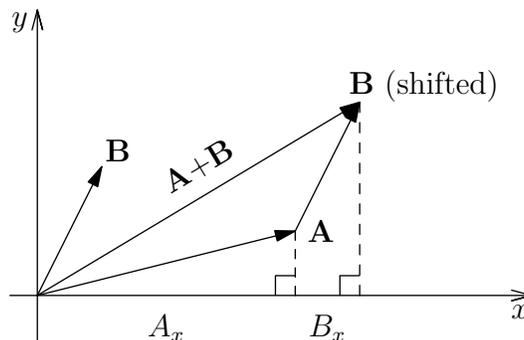
Using basic trigonometry,

$$v_x = v \cdot \cos \theta \quad (\text{the } x\text{-component of } \mathbf{v})$$

$$v_y = v \cdot \sin \theta \quad (\text{the } y\text{-component of } \mathbf{v})$$

Note from the figure that  $v$  (which is sometimes denoted explicitly by  $|\mathbf{v}|$ , which means the length of the vector  $\mathbf{v}$ ) is given by  $v^2 = v_x^2 + v_y^2$ , using the pythagorean theorem. In the example above,  $v = 50$  m/sec and  $\theta = 30^\circ$ , so that  $v_x = 43$  m/sec and  $v_y = 25$  m/sec. In this case, the  $x$ -component of  $\mathbf{v}$  is greater than the  $y$ -component of  $\mathbf{v}$  since the direction of  $\mathbf{v}$  is closer to the  $x$ -axis (east) than it is to the  $y$ -axis (north).

The easiest way to add two vectors is to add their  $x$  components to get a total  $x$  component, and separately do the same thing for the  $y$  components. Then, a new total vector can be made with the two total  $x$  and  $y$  components, using  $v_{\text{tot}}^2 = v_{x,\text{tot}}^2 + v_{y,\text{tot}}^2$  and  $\theta = \tan^{-1}(v_{y,\text{tot}}/v_{x,\text{tot}})$ . Graphically, this is the same as the “tip-to-tail” method, as in the figure below.



Here, vectors  $\mathbf{A}$  and  $\mathbf{B}$  are added by moving  $\mathbf{B}$  so that its tail is at the tip of  $\mathbf{A}$ , and then drawing the vector from the origin to the new tip of  $\mathbf{B}$ . It should be clear from the figure that the  $x$  components of  $\mathbf{A}$  and (the shifted)  $\mathbf{B}$  add up to the  $x$  component of the new vector, and similarly for the  $y$  components.

# SAT Subject Physics Facts & Formulas

## *Kinematics*

The following formulas for position  $x$ , velocity  $v$ , and acceleration  $a$  are valid when the acceleration of the object is constant. The initial value of a variable, such as position for example, is given by  $x_i$ , and the final value is given by  $x_f$ . The change in the variable, such as velocity for example, is given by  $\Delta v = v_f - v_i$ .

There are five main equations for kinematics which are all valid, but the one or two that you use will depend on the variable that you need and the information that you have.

When you don't have	Equation to Use
$a$	$\Delta x = v_{\text{ave}}\Delta t = \frac{1}{2}(v_i + v_f)\Delta t$
$\Delta x$	$\Delta v = a\Delta t$
$v_f$	$\Delta x = v_i\Delta t + \frac{1}{2}a(\Delta t)^2$
$v_i$	$\Delta x = v_f\Delta t - \frac{1}{2}a(\Delta t)^2$
$\Delta t$	$v_f^2 = v_i^2 + 2a\Delta x$

A note about graphs: the slope of a position vs. time graph is the velocity. Also, the slope of the velocity vs. time graph is the acceleration.

## *Dynamics*

Dynamics is the application of Newton's Laws to determine how a mass  $m$  moves when a force (or forces) is applied.

Newton's First Law is: *an object which moves at a constant velocity will continue moving at the same velocity unless it is acted upon by a non-zero force.* The force could be a single force, or several forces which are unbalanced (don't add to zero). Note that an object at rest has a constant velocity of zero, so it will remain at rest unless acted upon by such a force.

Newton's Second Law is: *the force on a mass equals the mass multiplied by the acceleration.* As a formula:

$$\mathbf{F} = m\mathbf{a}$$

where  $\mathbf{F}$  is the force vector,  $m$  is the mass, and  $\mathbf{a}$  is the acceleration vector. It is important to remember that the force in  $F = ma$  is the sum of all the forces (often called the "net force") acting on the mass, not just one particular force. The net force acting on a book

## SAT Subject Physics Facts & Formulas

resting on a table is zero: the weight of the book and the force of the table pushing up on the book add to zero. Note that the weight of an object (which is a *force*) due to gravity is:

$$W = mg$$

where  $W$  is the weight,  $m$  is the mass, and  $g$  is the acceleration due to gravity ( $g$  is approximately  $10 \text{ m/s}^2$  at or near the surface of the earth).

Friction is a force due to the contact of two rough surfaces against one another. The direction of the friction force is always opposite to the direction of motion (or, in the case of *static* friction, opposite to the motion that would occur if there were no friction). The magnitude of the friction force is proportional to the normal force holding the two surfaces together. The constant of proportionality is called the coefficient of friction, and is denoted by  $\mu$  (this symbol is the Greek letter mu). In formula form:

$$f = \mu N$$

where  $f$  is the friction force,  $\mu$  is the coefficient of friction, and  $N$  is the normal force. If there is no motion between the surfaces, the friction is static, and  $\mu = \mu_s$ , the static coefficient of friction. In the case of the two surfaces moving against one another,  $\mu = \mu_k$ , the kinetic coefficient of friction. Generally,  $\mu_s$  is larger than  $\mu_k$ ; however, the basic formula  $f = \mu N$  remains the same for both cases.

### Momentum

Momentum is defined to be the product of mass and velocity:

$$\mathbf{p} = m\mathbf{v}$$

where  $\mathbf{p}$  is the momentum,  $m$  is the mass, and  $\mathbf{v}$  is the velocity. Note that the momentum  $\mathbf{p}$  and velocity  $\mathbf{v}$  are both vectors, and they are in the same direction, since the mass  $m$  is just a positive number.

The net force  $F$  acting on a mass  $m$  for an amount of time  $\Delta t$  produces a change in momentum given by

$$\Delta p = F\Delta t.$$

The product  $F\Delta t$  is often called the *impulse*. Here, the change in momentum is just  $\Delta p = m\Delta v = m(v_f - v_i)$ , where  $v_i$  is the initial velocity and  $v_f$  is the final velocity.

Conservation of momentum: if there are no external forces on a system (or, the forces add to zero), then the momentum of a system is *conserved*, i.e., the momentum is constant. For example, consider when a rifle fires a bullet (in which case the “system” consists of the rifle plus the bullet). Before firing,  $p = 0$ . Since the external forces on the system add to zero (the weight of the rifle is balanced by the person holding it), then  $p = 0$  after firing, also. Therefore,  $m_b v_b + m_r v_r = 0$  and the recoil velocity of the rifle is  $v_r = -(m_b/m_r) \cdot v_b$ .

## SAT Subject Physics Facts & Formulas

In collisions, momentum is also conserved. For example, suppose two cars (masses  $m_1$  and  $m_2$ ) collide at velocities  $v_1$  and  $v_2$ . Then, their velocities after colliding ( $v'_1$  and  $v'_2$ ) satisfy the equation  $m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2$ . To solve a problem like this for the individual final velocities, more information must be given. For example, if the cars stick together, then the final velocity  $v_f$  can be found by solving  $m_1v_1 + m_2v_2 = (m_1 + m_2) \cdot v_f$  for  $v_f$ .

### *Work, Energy, and Power*

Work has a very specific meaning in physics. Work is done when a force is applied to an object as it moves a distance. The amount of work done is given by:

$$W = Fd \cos \theta$$

where  $W$  is the work done, in joules (1 J = 1 newton meter),  $d$  is the distance, and  $\theta$  is the angle between the direction of the force and the direction of motion. If the force is in direction of motion,  $\theta = 0^\circ$ , so  $\cos \theta = 1$  and the work is just force times distance,  $W = Fd$ .

If there is no displacement at all (i.e.,  $d = 0$ ), then no work is done. Also, if the force is at right angles to the direction of motion ( $\theta = 90^\circ$ ), again no work is done, since  $\cos 90^\circ = 0$ . Note that work can be negative, for example, if  $\theta = 180^\circ$ , then  $W = -Fd$  and  $W < 0$  ( $\cos 180^\circ = -1$ ). The work done by kinetic friction, for example, is always negative since the force of friction is always directed opposite to the direction of motion.

The *energy* of an object can be thought of as the ability of that object to do work. Or, conversely, work must be done by or on an object to change the object's energy. There are two main kinds of energy. The first is called *kinetic* energy and is associated with objects that are moving ( $v \neq 0$ ). The amount of kinetic energy is given by

$$\text{KE} = \frac{1}{2}mv^2$$

where  $m$  is the mass and  $v$  is the velocity. For example, when someone throws a 0.5 kg softball, the softball goes from rest ( $v = 0$ ) to some velocity (say, 8 m/s). The kinetic energy of the softball has increased from zero to  $1/2 \cdot 0.5 \text{ kg} \cdot 8 \text{ m/s}^2 = 16$  joules, so the thrower has done +16 joules of work on the softball. When the ball is caught, the catcher must do (negative) work on the ball (namely, -16 J) to change its energy from 16 joules back to zero. Equivalently, the ball will do 16 joules of work on the catcher as it comes to a halt.

The second main kind of energy is called *potential* energy. This energy is associated with the position of the object (for example, the height of a mass measured from the ground below) or its configuration (for example, a compressed spring). In the case of a mass  $m$  at a height  $h$ , the potential energy is

$$\text{PE} = mgh.$$

Just as before, work must be done on or by the object to change its energy. In this case, perhaps the mass was carried by someone on the ground up to the height  $h$ , at a constant

## SAT Subject Physics Facts & Formulas

velocity. The work done by the person is the force applied ( $mg$ , to counteract gravity) times the distance ( $h$ ), which gives  $W = mgh$ . This work is “stored” as potential energy in the mass at height  $h$ .

The mass now has the ability to do work. For example, if the mass is a large boulder at the top of a cliff, the boulder could be used to make a crater in the ground below (by pushing it off the cliff). The amount of work done by the boulder on the ground would be  $W = mgh$ .

We can combine the potential and kinetic energy to get the total energy:

$$E = \text{KE} + \text{PE}.$$

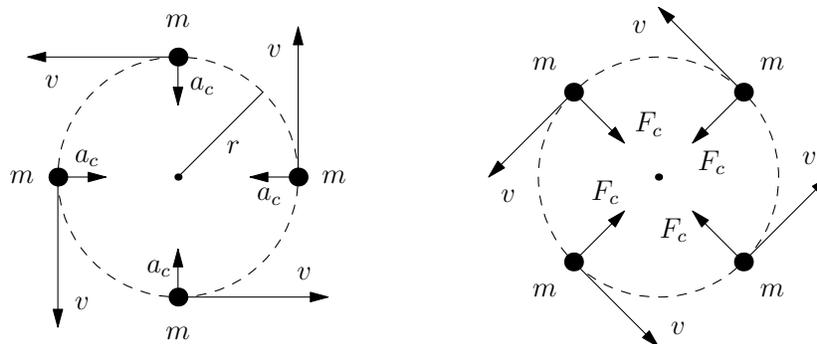
In the absence of forces such as friction, the total energy of an object remains constant (is *conserved*). For example, suppose the boulder from before is 100 kg and is on a cliff 10 m high. The total energy of the boulder is  $E = \text{KE} + \text{PE} = 0 + (100 \text{ kg})(10 \text{ m/s}^2)(10 \text{ m}) = 10000 \text{ J}$ . Now the boulder is pushed off the cliff. Just before it hits the ground,  $E$  is still 10000 J. But now,  $\text{PE} = 0$  and  $\text{KE} = 10000 \text{ J}$ . Halfway down the cliff,  $\text{PE} = (100 \text{ kg})(10 \text{ m/s}^2)(5 \text{ m}) = 5000 \text{ J}$ . Since  $E$  is still 10000 J, then we know that  $\text{KE} = 5000 \text{ J}$ . In each case, the velocity could be found by using  $\text{KE} = 0.5 \cdot m \cdot v^2$ .

*Power* is the rate at which work is done: if a certain amount of work  $W$  is done in an elapsed time  $\Delta t$ , the power is

$$P = \frac{W}{\Delta t}.$$

For example, a 50-watt incandescent light bulb uses 50 joules of energy for each second that it is turned on. A 100-watt bulb uses that same amount of energy in only 0.5 seconds, since  $P = 100 \text{ W} = 50 \text{ J}/0.5 \text{ sec}$ . (In both cases, about 90% of the work being done by the light bulbs goes into heat, not light! Fluorescent and LED lights are much more efficient.)

### Circular Motion



When an object moves in a circle with a constant speed, the object is said to be in *uniform circular motion*. In the figure above, an object of mass  $m$  is moving uniformly in a circle of radius  $r$ , counter-clockwise, and the position of the mass is shown at eight different points

## SAT Subject Physics Facts & Formulas

on the circle. This mass is under constant acceleration, not because the speed is changing (it isn't) but because the *direction* of the velocity is changing (remember that velocity is a vector, and so it has both a magnitude (speed) and a direction).

In the figure above, the force on the mass ( $F_c$ ) along with its acceleration ( $a_c$ ) are shown at eight snapshots in time during the motion of the mass around the circle. Both  $F_c$  and  $a_c$  always point directly toward the center of the circle, so in either circle in the above figure,  $a_c$  could be replaced by  $F_c$ , and vice versa.

The acceleration  $a_c$ , called “centripetal” acceleration, is given by

$$a_c = \frac{v^2}{r}.$$

The corresponding centripetal force, which keeps the mass  $m$  going around in the circle, is

$$F_c = \frac{mv^2}{r}.$$

The period  $T$  is the time it takes for the object to make one revolution. Given  $T$ , the velocity can be found using

$$v = \frac{2\pi r}{T}.$$

Here,  $2\pi r$  is the distance the object goes in one revolution and  $T$  is how long it took to go that distance. Sometimes, instead of  $T$ , the frequency  $f$  is given, where  $f$  is the number of revolutions per second. These two numbers are related by

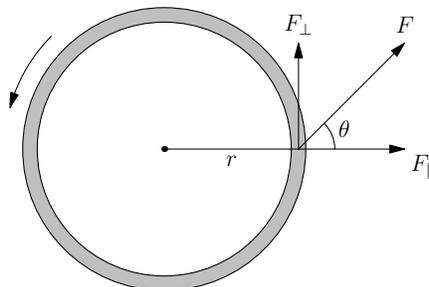
$$f = 1/T.$$

### *Torque And Angular Momentum*

Torque is a force ( $F$ ) that tends to make an object rotate. To do that, the force must act at a distance ( $r$ ) from an axis of rotation:

$$\text{torque} = rF_{\perp}.$$

Here,  $F_{\perp}$  is the component of the force  $F$  that is perpendicular to the direction of the radius. In the figure below, a force  $F$  is applied to an object (a wheel, say). The component of the force  $F$  that is parallel to the radius ( $F_{\parallel}$ ) can't make the wheel rotate, and so it doesn't contribute to the torque.



## SAT Subject Physics Facts & Formulas

If  $\theta$  is the angle between the radial direction and  $F$ , then  $F_{\perp} = F \sin \theta$ , so that the torque equation can also be written as: torque =  $rF \sin \theta$ . The perpendicular force  $F_{\perp}$  will tend to rotate the wheel in the figure in the counter-clockwise direction.

Angular momentum is the circular equivalent of linear momentum ( $p = mv$ ), and is given by:

$$L = mvr.$$

For example,  $L = mvr$  is the angular momentum of the rotating object in the diagram at the beginning of the section on circular motion.

Similarly to regular (linear) momentum, if there are no external torques on a system (or, the torques add to zero), then the angular momentum of a system is *conserved*, i.e., the angular momentum is constant. For example, could the Earth just stop spinning in the middle of the night? The biggest force on the Earth is due to the Sun (see the Gravity section below), but that force effectively acts at the center of the Earth, where  $r = 0$ . This means that the torque due to the force is zero, so there are no external torques on the Earth, and therefore it will just keep on spinning.

A little more complicated example is the system of the Earth and the Moon. The Moon (which has a large mass) moves roughly in a circle about the Earth, and so it has angular momentum. The Earth has angular momentum since it is spinning about its own axis once per day (and this includes *you*, since you are on the surface of the Earth, rotating about the axis). The Moon causes the tides on the Earth, but the drag of the sea sloshing about is gradually slowing down the spin rate of the Earth. The conservation of angular momentum in the Earth-Moon system implies that the angular momentum of the moon must be increasing, namely, the  $r$  in the angular momentum ( $mvr$ ) of the Moon must be getting bigger. In fact, the distance to the moon has been measured to be increasing at roughly 4.5 cm per year.

### *Springs*

A spring is a metal coil which, when stretched, pulls back on the object attached to the end of the spring. When compressed, the spring pushes against the object at the end of the spring. When not stretched or compressed, the spring is at its “natural length” and it doesn’t exert a force on the object at all.

The restoring force  $F_s$  of a spring is proportional to the amount (distance) that the spring is stretched or compressed. If this distance is  $x$ , then the restoring force is

$$F_s = kx.$$

The formula above is often called “Hooke’s Law”. When a spring is stretched or compressed, it has a (stored) potential energy of

$$\text{PE}_s = \frac{1}{2}kx^2.$$

# SAT Subject Physics Facts & Formulas

## Gravity

Gravity is a force that occurs between objects that have mass. If two masses  $m_1$  and  $m_2$  are separated by a distance  $r$ , then the force of gravity between them is

$$F_g = G \frac{m_1 m_2}{r^2}$$

where  $G$  is just a number which is always the same for every calculation, i.e.,  $G$  is a constant. In metric units, the number turns out to be  $6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$ , but it isn't important to know this particular number.

The above mass pairs could be everything from two billiard balls (same mass) to you (very small mass) and the earth (very large mass). Notice that the force of gravity is inversely proportional to the distance of separation, and proportional to the product of the two masses. For example, if the distance between  $m_1$  and  $m_2$  were to double, then the force would be only 25% as large. If the mass of the earth were doubled, the force on you (for example) would become twice as big, i.e., you would weigh twice as much as you do now!

## Electric Fields, Forces, And Potentials

Charge is to electric force as mass is to gravitational force. However, unlike mass, charge comes in two types: positive, and negative. Like charges ( $++$  or  $--$ ) repel, and unlike charges ( $+-$ ) attract. The force of attraction or repulsion is given by "Coulomb's Law":

$$F_e = k \frac{q_1 q_2}{r^2}$$

where  $r$  is the distance between the two charges  $q_1$  and  $q_2$  and  $k$  is a constant (about  $9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$ ). Notice how similar this force law is to force law for gravitation (see the Gravity section). Charge is measured in coulombs (denoted as: "C"). To get an idea of what a coulomb of charge is, a typical 100 watt light bulb has about 1 C of charge passing through it each second.

Instead of one charge attracting or repelling another, we can think of either charge as generating an *electric field*. Then, any other charge, when placed in this field, will feel a force (again, either attractive or repulsive). The electric field is defined to be the force on the second charge (often called a *test* charge) divided by the test charge amount. If the first charge is  $q$  and the test charge  $q_t$  feels a force  $F_t$ , then the electric field produced by  $q$  is:

$$E = \frac{F_t}{q_t}$$

Using Coulomb's Law, with  $q_1 = q$  and  $q_2 = q_t$ , then we see that the electric field due to  $q$  is:

$$E = k \frac{q}{r^2}$$

## SAT Subject Physics Facts & Formulas

Electric fields can also be set up by, for example, two parallel, flat, conducting (metal) plates. When these plates are connected to a battery of voltage  $V$ , one plate to each terminal, a uniform electric field is generated in between the plates:

$$E = \frac{V}{d}$$

where  $d$  is the distance of separation between the plates. The electric field points from the plate connected to the battery's positive terminal to the other plate, and shows which way a positive test charge would move if placed in that field (i.e., away from the positive plate!).

### *Circuits*

In a circuit, a certain amount of charge per unit time, called the current (denoted by  $I$ ), flows past any given point in the circuit. The voltage  $V$ , say due to a battery in the circuit, is equal to the work done by the battery per unit charge. These two quantities, voltage and current, are proportional to one another, as related by "Ohm's Law":

$$V = IR.$$

The constant of proportionality  $R$  is called the resistance in the circuit. For a given voltage  $V$ , a low resistance corresponds to a high current and a high resistance corresponds to a low current, since  $I = V/R$ . An analogy can be made to a waterfall: the current in a circuit is just like the amount (current) of water falling down, and the voltage in a circuit is similar to the height of the waterfall. The voltage can be high with low current (imagine a tall waterfall with just a trickle of water), or the voltage can be low with high current (a short waterfall with a large current of water), depending on the resistance (kind of similar to rocks in the stream).

The power, or energy per unit time, dissipated in a resistor  $R$  is:

$$P = IV = \frac{V^2}{R} = I^2R,$$

where Ohm's Law was used to substitute for  $I$  or  $V$  in  $P = IV$ . You can use whichever formula requires the quantities that you are given.

If two or more resistors are placed in series, the total resistance is the sum:

$$R_s = R_1 + R_2 + \dots,$$

where  $R_s$  is called the *series* resistance. Note that the series resistance is always bigger than the biggest of the individual resistances.

If two or more resistors are placed in parallel, the total resistance is the inverse of the sum of the inverses:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \dots,$$

## SAT Subject Physics Facts & Formulas

where  $R_p$  is called the *parallel* resistance. Note that the parallel resistance is always less than the smallest of the individual resistances.

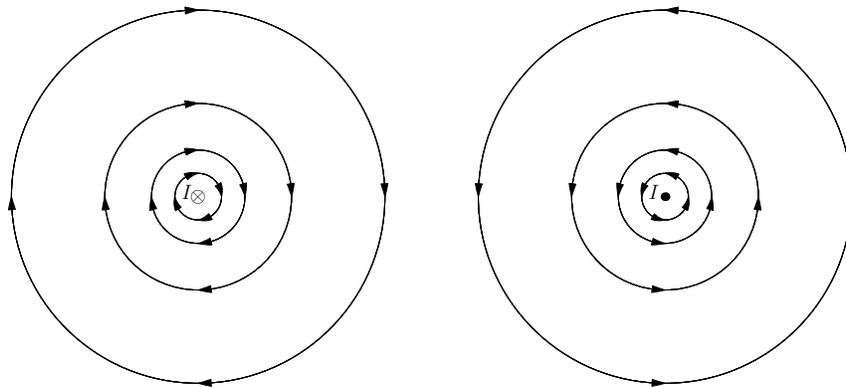
A capacitor is a circuit element made up of two metal plates held a small distance apart. When connected to a battery providing a voltage  $V$ , charge builds up on the plates, positive on one plate, and negative on the other. The amount of charge on either plate is proportional to the voltage:

$$Q = CV,$$

where  $C$  is called the capacitance. The capacitance depends on the size of the plates and how far apart they are, similar to how the resistance of a resistor depends on the length and width of the material in it. Unlike a resistor, however, charge doesn't actually flow *through* a capacitor; instead, charge either builds up on it, or flows out from it, depending on whether a battery is connected or disconnected.

### *Magnetic Fields and Forces*

A magnetic field (usually denoted by  $B$ ) is produced by charges that are in motion ( $v \neq 0$ ). The current in a wire, for example, is due to the motion of charge, and so a current-carrying wire produces a magnetic field. In the diagram below, there are two wires, each perpendicular to the page (or screen). The current in the wire on the left is moving into the page (denoted by the symbol  $\otimes$ ), whereas the current in the wire on the right is coming directly out of the page (denoted by the symbol  $\bullet$ ). The magnetic field makes circles around each wire, in the direction shown by the arrows. The field makes circles around the wire: if you point the thumb of your right hand in the direction of the current, the magnetic field is in the direction of your curled fingers (this recipe is called the “right-hand rule”).



The magnitude of the magnetic field around a long wire is inversely proportional to the distance from the wire. In the diagram, this is denoted by drawing the field lines closer together near the wire.

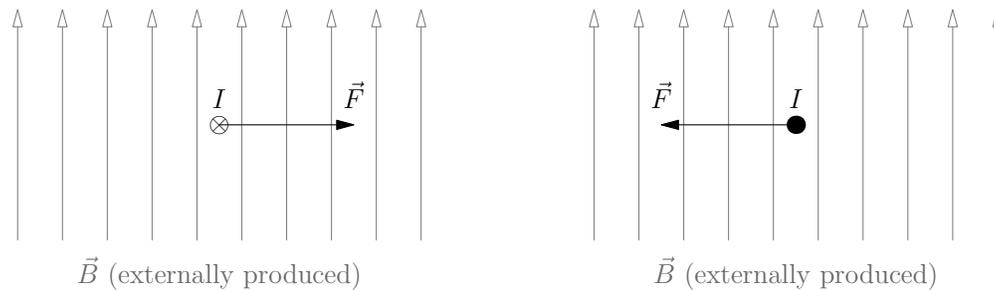
Not only do current-carrying wires produce a magnetic field, they also feel a force when placed within *another* magnetic field (being produced by something else, perhaps another current-carrying wire). The magnitude of the force depends on how much current is in

## SAT Subject Physics Facts & Formulas

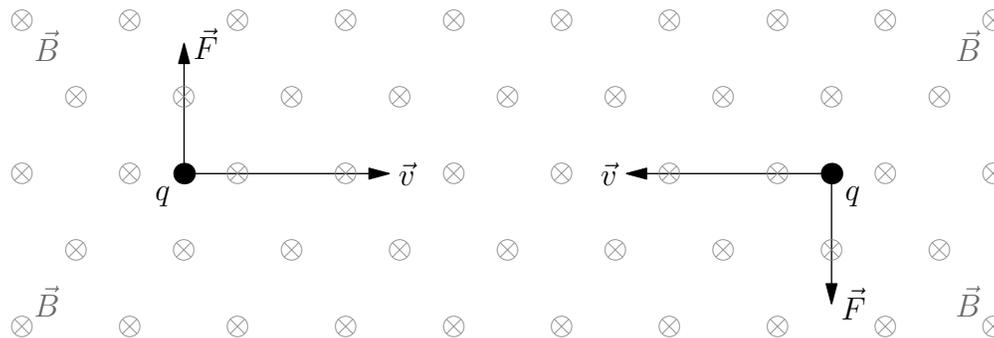
the wire, how strong the magnetic field is, and how long the wire is. More precisely, the magnitude of the force is

$$F = ILB \sin \theta$$

where  $I$  is the current in the wire,  $B$  is the magnetic field,  $L$  is the length of the wire, and  $\theta$  is the angle between the direction of the current and the magnetic field direction. Usually for the SAT Subject physics test,  $\theta = 90^\circ$  so that  $\sin \theta = 1$  and  $F = ILB$ . The direction of the force comes from the “second right-hand rule”: if you point your thumb in the direction of the current and point your fingers in the direction of the magnetic field, the force on the wire comes out of your palm.



In the above diagram, a uniform magnetic field is being externally produced and points up. The current in the wire on the left is into the page (so that  $\theta = 90^\circ$ ), and the force on the wire (due to the magnetic field that is being applied to it) is directed to the right. The current in the wire on the right is out of the page, and so the force on the wire is directed to the left. See if you can use the second form of the right-hand rule to verify the directions of the forces shown in the diagram.



The situation is very similar for a single point charge moving in a magnetic field. In the diagram above, the magnetic field is everywhere pointing into the page; a positive charge ( $q > 0$ ) on the left is moving with speed  $v$  to the right (so that  $\theta = 90^\circ$ ), and another positive charge on the right is moving to the left with the same speed. In general, the magnitude of the force on the charge is

$$F = qvB \sin \theta$$

where  $q$  is the charge,  $B$  is the magnetic field,  $v$  is the magnitude of the velocity of the charge, and  $\theta$  is the angle between the direction of travel of the charge and the magnetic

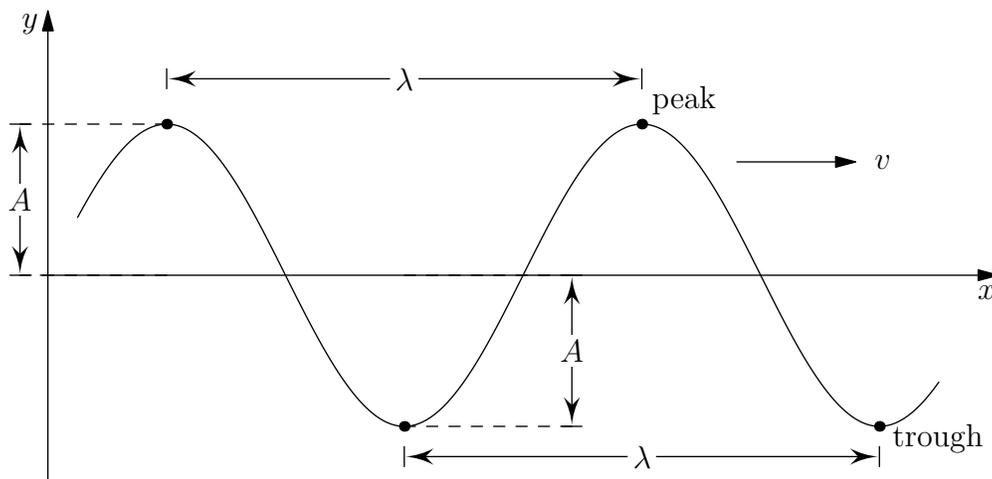
## SAT Subject Physics Facts & Formulas

field direction. Again, the typical situation for the SAT Subject test in physics is  $\theta = 90^\circ$  so that  $\sin \theta = 1$  and  $F = qvB$  (as in the diagram). You can use the same “second right-hand rule” used to determine the direction of the force on the wire, except that the thumb now points in the direction that the charge is moving. See if you can verify the directions of the forces shown in the diagram. Note that the charges in the diagram are positive; if instead the charges were negative ( $q < 0$ ), the forces would be in the opposite direction from that given by the right-hand rule.

Also, note that a charge at rest, no matter how large, will not feel a force from a magnetic field, since  $v = 0$  and so  $F = 0$ . If you are a very observant reader, you may wonder how this can be: doesn't the measured velocity of an object depend on who is doing the measurement? Suppose you are standing next to a road, with a charge  $q$  at rest in your hand, in the middle of a uniform magnetic field. Since  $v = 0$ , you will say that the charge feels no force. However, for someone else who is driving by in a car at a constant velocity, say, the charge *is* moving in a magnetic field (with a speed equal to that of the car), so it seems that one person (the one in the car) will say there is a force and another person (you) will say that there isn't. Which person is correct? (This is a tough question: think about it, and then ask your physics teacher.)

### *Waves and Sound*

Waves are the transfer of energy from one point to another, and are usually one of two different kinds: mechanical, or electromagnetic. A mechanical wave travels through a medium such as air, water, or solid material. For example, when two people are talking, sound waves are traveling through the air between them. A tsunami is a mechanical wave traveling through the ocean. The figure below shows an example of a wave; one can imagine it to be the view of the surface of the ocean from the side as the wave passes. A boat on the surface of the water would bob up and down as it rides the wave.



In contrast to mechanical waves, electromagnetic waves can travel with or without a medium. Familiar examples include light and energy from the sun, radio waves, and the microwave signals sent from GPS satellites to a portable navigation device or mobile

## SAT Subject Physics Facts & Formulas

phone. Electromagnetic waves are *transverse* waves, in which the vibrations occur in a direction perpendicular to the direction in which the wave is travelling. Another example of transverse waves are the waves that travel down a plucked guitar string. However, the sound waves produced in the air by that guitar string are examples of *longitudinal* waves: the direction of the sound wave is parallel to the vibrations of the wave.

The diagram at the beginning of the section shows the important features of a transverse wave. In the diagram, the wave is moving in the  $x$  direction and the vibrations are occurring in the  $y$  direction. The distance from one peak to the next (or from one trough to the next) is called the wavelength, and is denoted by the greek letter lambda ( $\lambda$ ). The amplitude ( $A$ ) is the maximum displacement of the wave, measured at the peak (or at the trough). The frequency ( $f$ ) of the wave is the number of peaks passing a given point (say,  $x = 0$ ) per second. The frequency of the wave is related to its wavelength by the “wave speed equation”:

$$v = \lambda f,$$

where  $v$  is the speed of the wave. If you know any two of the quantities  $f$ ,  $\lambda$ , or  $v$ , then you can determine the third. Also, the equation shows that, for a given wave speed, a long wavelength corresponds to a small frequency, and a short wavelength corresponds to a large frequency. On the SAT Subject test in physics, a commonly asked type of question includes the following: “If the speed of a wave is doubled, and the frequency is halved, how does the new wavelength compare to the old wavelength?” You should be able to show using the wave speed equation that  $\lambda$  is quadrupled compared to the original value.

The period ( $T$ ) of the wave is the time it takes for one complete wave vibration (or “cycle”) to pass a given point (say,  $x = 0$ ):  $T$  is given by  $T = 1/f$ . For example, suppose someone with a watch in the aforementioned boat notices that 3 seconds elapse between when the boat is at the highest points of the wave (the peaks). Since one complete wave cycle is between two peaks, this means that there is one wave cycle per 3 seconds, i.e., the frequency is  $f = 1/3$  Hz. Another way of understanding the wave speed equation is to realize that the wave travels a distance  $\lambda$  in the amount of time  $T$  (since a complete vibration corresponds to that distance). So, the speed of the wave is just  $v = \lambda/T = \lambda f$ .

Sound is a mechanical wave: it travels through a medium such as air, water, or even solid materials. The frequency of sound is what we perceive as pitch, or the “highness” or “lowness” of the sound. The amplitude is related to how loud the sound is, in that more amplitude means a louder sound, and less amplitude means a quieter sound. The speed of the sound wave depends on the medium it is traveling through, among other things. For example, the speed of sound in air is about 340 m/s (1100 ft/s), but the speed of sound in water is much faster: about 1500 m/s (4900 ft/s). The sound of someone speaking typically has a frequency of 500 Hz, which gives a wavelength in air of  $\lambda = v/f = 0.7$  m (2.2 ft).

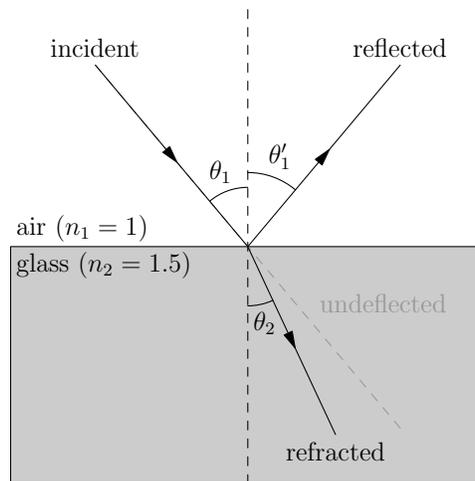
When the source of a wave is itself moving, the Doppler effect occurs. If the source is moving toward you, the frequency (pitch) is higher than it normally is. If the source is moving away from you, the frequency is lower than it normally is. For example, the siren

## SAT Subject Physics Facts & Formulas

on an ambulance going by you on its way to the hospital sounds higher pitched coming toward you and lower pitched going away.

### *Light and Optics*

Light is an electromagnetic wave, with a frequency ( $f$ ) and wavelength ( $\lambda$ ) just like any other wave. In a vacuum, light travels at  $3 \times 10^8$  m/s (670 million miles per hour), which is fast enough to circle the earth more than 7 times in one second. The speed of light is usually denoted by  $c$ , so that  $\lambda f = c$ . Different frequencies (or wavelengths) of light correspond to the various colors that we see, everything from violet at lower wavelengths ( $\lambda \approx 400 \times 10^{-9}$  m) to green in the middle ( $\lambda \approx 550 \times 10^{-9}$  m) to red at the higher wavelengths ( $\lambda \approx 700 \times 10^{-9}$  m).



When light bounces (reflects) off a shiny surface, the angle of incidence equals the angle of reflection. By convention, the angle is measured with respect to the “normal” (a line drawn perpendicular) to the reflective surface. In the diagram above, the normal is the dashed vertical line in the middle, and the law of reflection tells us that  $\theta_1 = \theta'_1$ . When light travels from one medium (say, air) to another (say, glass), it bends (refracts). This is due to the fact that light travels at different speeds in different media. In any medium other than vacuum, light travels more slowly than  $3 \times 10^8$  m/s. Specifically,

$$v = c/n$$

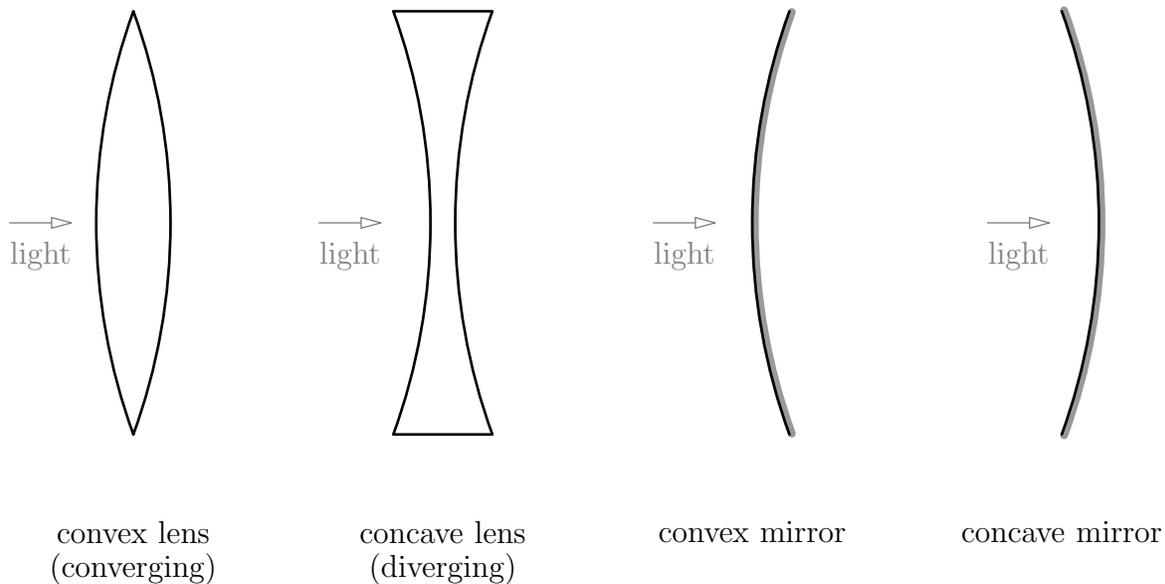
where  $v$  is the speed of light in the medium,  $c$  is the speed of light in vacuum, and  $n$  is a number called the “index of refraction” for the medium. For example, the index of refraction for typical glass is  $n = 1.5$ , which means that the speed of light in glass is about  $(3 \times 10^8 \text{ m/s})/1.5 = 2 \times 10^8$  m/s. The incident angle and refracted angle (again, measured from the normal) are related by “Snell’s Law”:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2,$$

## SAT Subject Physics Facts & Formulas

where  $n_1$  and  $\theta_1$  refer to the media from which the light is coming and  $n_2$  and  $\theta_2$  refer to the media into which the light is travelling.

A thin lens is a thin piece of material (often, glass) that bends (refracts) light as described by Snell's Law. A mirror is a reflective, and sometimes curved, surface that reflects light so that the angle of incidence equals the angle of reflection. Every thin lens (or mirror) can be described by one number called the focal length ( $f$ , not to be confused with frequency). When an object is far away from a lens, the light from the object as it strikes the lens is essentially a group of parallel rays. The figure below shows what happens when light from a distant object strikes various optical instruments: converging and diverging lenses, and concave and convex mirrors.



For a converging lens or concave mirror,  $f$  is the distance from the lens or mirror to the place where these rays will converge after focussing. An image is formed where the light rays converge after going through the lens or getting reflected from the mirror. A diverging lens and convex mirror make the parallel light rays diverge, so that no real image is formed. If  $d_o$  is the distance from the object to the lens, and  $d_i$  is the distance from the lens to the image, then

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

relates these distances to the focal length of the mirror or lens. When using this formula (called the “thin-lens” formula), the signs of the quantities involved are very important.

For converging lenses or concave mirrors,  $f > 0$ , whereas for a diverging lenses and convex mirrors,  $f < 0$ . If the resulting image distance  $d_i$  is negative, the image is not real (the light wasn't focussed). We can see these “virtual” images nonetheless because our eyes are fooled into thinking that the diverging rays are being produced by an image located

## SAT Subject Physics Facts & Formulas

at a distance  $d_i$  from the lens. The object distance  $d_o$  is always positive for a single lens or mirror because the object is always real.

We are often interested in knowing how big the image will be compared to the object, and whether it will be upright or inverted. Both size and orientation are given by

$$m = -\frac{d_i}{d_o}$$

where  $d_i$  and  $d_o$  are the image and object distances as usual (signs included). The sign of  $m$  tells us the orientation of the image compared to the object: If  $m < 0$ , the image is inverted compared to the object; if  $m > 0$ , the image is the same orientation as the object. Since  $d_o > 0$ , the magnification formula suggests that virtual images ( $d_i < 0$ ) are upright compared to the object, whereas real images ( $d_i > 0$ ) are inverted.

The absolute value of the magnification describes how big the image is compared to the object, i.e.,

$$|m| = \frac{\text{image size}}{\text{object size}}.$$

Here, the image can be bigger ( $|m| > 1$ ) or smaller ( $|m| < 1$ ) than the object size.

Type of Lens/Mirror	Object Distance	Image Distance	Real Image?	Upright Image?	Image Size
concave	$d_o > 2f$	$f < d_i < 2f$	yes	no	smaller
mirror	$d_o = 2f$	$d_i = 2f$	yes	no	same
or	$f < d_o < 2f$	$d_i > 2f$	yes	no	bigger
converging	$d_o = f$	no image	—	—	—
lens	$d_o < f$	$d_i > d_o$	no	yes	bigger
diverging or convex lens/mirror	any	$d_i < d_o$	no	yes	smaller

The table above shows the different image outcomes that occur depending the object's distance from the lens. You do *not* need to memorize this table; however, see if you can verify each row of the table using the lens formula. For example, let  $f = 50$  cm. If  $d_o = 50$  cm, the lens formula tells us that  $1/d_i = 0$ , so that the image distance is at infinity (i.e., no image is formed). Check the second row of the table by setting  $d_o = 100$  cm.